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1987 J. Phys. A: Math. Gen. 20 5361

(http://iopscience.iop.org/0305-4470/20/15/047)

J. Phys. A: Math. Gen. 20 (1987) 5361-5369. Printed in the UK

Finite-size effects and infrared asymptotics of the correlation functions in two dimensions

N M Bogoliubov, A G Izergin and N Yu Reshetikhin

Leningrad Branch of Steklov Mathematical Institute, Fontanka 27, Leningrad, 191011, USSR

Received 2 February 1987

Abstract. The finite-size corrections approach to the calculation of the asymptotics of correlation functions containing the oscillating terms in the asymptotics in (1+1)-dimensional theories is presented. Explicit formulae for the critical exponents in integrable models are obtained.

1. Introduction

Phase transitions in quantum models in two spacetime dimensions take place only at zero temperature. This means that the exponential form of the long distance asymptotics of correlation functions changes for the power form. These systems have a gapless excitation spectrum with a linear dispersion law in the vicinity of the Fermi level. Their critical behaviour is described by conformal field theory [1, 2] which is parametrised by the central charge c in the Virasoro algebra satisfied by the energy-momentum tensor. For $c<1$ a discrete set of values of c is allowed by unitarity [3]: $c=1-6/m(m+1)$ ($m\geqslant 2$ an integer) and the scaling dimensions are known exactly [2]. When $c\geqslant 1$ the critical exponents may continuously depend on the parameters of the model [4]. To obtain the complete information about the critical behaviour of the system one has to determine the central charge and the conformal dimensions Δ, $\bar{\Delta}$ of the primary fields [1, 2]. These quantities can be calculated from the leading finite-size corrections to the ground and the excited energies of the system [1, 5].

Let us consider the system in a periodic box of length L. Let E_L^0 be the energy of the ground state $|0\rangle$ and E_L^ϕ the minimal energy of the excited state $|\phi\rangle$ with the non-vanishing form factor $\langle 0|\phi(0)|\phi\rangle\neq 0$. Suppose that in the limit $L\to\infty$ one has

$$E_L^0 = Lf_0-(\pi c/6v)/L+\mathrm{O}(1/L^2) \tag{1}$$

$$E_L^\phi - E_L^0 = 2\pi v\theta_\phi/L \tag{2}$$

$$P_L^\phi = 2\pi s_\phi/L. \tag{3}$$

Here P_L^ϕ is the momentum of the state $|\phi\rangle$ and v is the Fermi velocity. Then the leading term of the long distance asymptotics of the correlation function is of the following conformal form:

$$\langle\phi(z,\bar{z})\phi(0,0)\rangle = \langle\phi(0,0)\rangle^2 + A/(z^{2\Delta_\phi}\bar{z}^{2\bar{\Delta}_\phi}) \tag{4}$$

where $\Delta_\phi = (\theta_\phi + s_\phi)/2$, $\bar{\Delta}_\phi = (\theta_\phi - s_\phi)/2$ and $z = v\tau + \mathrm{i}x$, $\bar{z} = v\tau - \mathrm{i}x$. The amplitude here is

$$A = \lim_{L\to\infty} \{(\pi/L)^{-2\theta_\phi} \exp(\mathrm{i}\pi s_\phi)|\langle 0|\phi(0)|\phi\rangle|^2\}.$$

This means that for $|z|\to\infty$ the operator $\phi(z, \bar{z})$ is a conformal one with conformal dimension Δ_ϕ, $\bar{\Delta}_\phi$.

The central charge in the Virasoro algebra may also be calculated from the low-temperature expansion of the bulk free energy [6, 7]:

$$f(T) = f_0 - (\pi c/6v)T^2 + \mathrm{O}(T^2) \tag{5}$$

where T is the temperature.

The generalisation of the finite-size corrections approach to the calculation of the asymptotics of correlation functions containing oscillating terms in the asymptotics is presented below. This paper is an extended version of a previous paper of ours [8]. In § 2 the general description of the leading terms of the asymptotics is given. Explicit formulae for the critical exponents in integrable models are obtained. As an example, the one-dimensional Bose gas and the XXZ Heisenberg antiferromagnetic chain are considered in § 3.

2. Finite-size effects

Generally, the leading term of the correlation function asymptotics can oscillate, so the asymptotics is not always conformal. The reason is that the large distance asymptotics of the correlation functions is determined by processes of two kinds in the intermediate states (we consider the one-dimensional models). The processes of the first kind are transitions in the neighbourhood of the Fermi level: $\pm k_{\mathrm{F}} \to \pm k_{\mathrm{F}}$, $p \simeq 0$. Here p is the momentum of elementary excitation. The processes of the second kind are transitions from one Fermi level to another one: $\pm k_{\mathrm{F}} \to \mp k_{\mathrm{F}}$, $p \simeq \pm 2k_{\mathrm{F}}$.

Thus we have [9-11]

$$\langle\phi(x, 0)\phi(0, 0)\rangle - \langle\phi(0, 0)\rangle^2 \simeq \frac{A}{x^{2\theta_0}} + \frac{B\cos(2k_{\mathrm{F}}x)}{x^{2\theta_1}}. \tag{6}$$

The first and the second terms on the right-hand side are due to processes of the first and the second kind, respectively. If $\theta_0 < \theta_1$, then the first non-oscillating term in the asymptotics is the leading one. If $\theta_0 > \theta_1$, then the second term prevails and the asymptotics oscillates.

Equation (6) describes the equal-time correlation function and can be interpreted in the following way. For $x\to\infty$ the field ϕ can be decomposed into the sum of conformal fields which are responsible for the power asymptotics. Consider fields with the following expansion:

$$\phi(z, \bar{z}) \simeq \phi_0(z, \bar{z}) + \tilde{\phi}_0(z, \bar{z}) + \phi_1(z, \bar{z})\exp(\mathrm{i}k_{\mathrm{F}}x) + \phi_{-1}(z, \bar{z})\exp(-\mathrm{i}k_{\mathrm{F}}x). \tag{7}$$

The fields ϕ_0, $\tilde{\phi}_0$ and $\phi_{\pm 1}$ are conformal fields. The oscillations in (7) are due to macroscopic gaps in the momentum operator spectrum for processes of the second kind ($p \simeq \pm 2k_{\mathrm{F}}$).

The general form of the time-dependent correlation function asymptotics which corresponds to equation (7) is

$$\langle\phi(z,\bar z)\phi(0,0)\rangle \simeq \langle\phi(0,0)\rangle^2 + \frac{A}{z^{2\Delta_0}\bar z^{2\bar\Delta_0}} + \frac{\tilde A}{z^{2\tilde\Delta_0}\bar z^{2\bar{\tilde\Delta}_0}} + \frac{B\cos(2k_{\mathrm F}x)}{z^{2\Delta_1}\bar z^{2\bar\Delta_1}}. \tag{8}$$

We require that the conformal dimensions of the fields ϕ_1, ϕ_{-1} are the same and that the following orthogonality holds:

$$\langle\phi_0\tilde\phi_0\rangle = \langle\phi_1\phi_{-1}\rangle = \langle\phi_0\phi_{\pm1}\rangle = \langle\tilde\phi_0\phi_{\pm1}\rangle = 0.$$

Extending the methods of [5] we can deduce the scaling dimensions in (3) from the finite-size effects. The correlation function in the periodic strip of width L ($k_{\mathrm F}^{-1} \ll x \ll L$) is found with the use of the conformal transformation $w = (L/2\pi)\log z$ of the infinite plane result (8) (the variable x in the cosine remains unchanged):

$$\begin{aligned}\langle\phi(z,\bar z)\phi(0,0)\rangle_L &\\ \simeq \langle\phi(0,0)\rangle_L^2 &+ \frac{A(\pi/L)^{2(\Delta_0+\bar\Delta_0)}}{[\sinh(\pi z/L)]^{2\Delta_0}[\sinh(\pi\bar z/L)]^{2\bar\Delta_0}}\\ &+ \frac{\tilde A(\pi/L)^{2(\tilde\Delta+\bar{\tilde\Delta}_0)}}{[\sinh(\pi z/L)]^{2\tilde\Delta_0}[\sinh(\pi\bar z/L)]^{2\bar{\tilde\Delta}_0}} + \frac{B(\pi/L)^{2(\Delta_2+\bar\Delta_1)}\cos(2k_{\mathrm F}x)}{[\sinh(\pi z/L)]^{2\Delta_1}[\sinh(\pi\bar z/L)]^{2\bar\Delta_1}}.\end{aligned} \tag{9}$$

It can also be represented as a sum over the intermediate states:

$$\langle\phi(z,\bar z)\phi(0,0)\rangle_L = \sum_n |\langle 0|\phi(0,0)|n\rangle|^2 \exp[-\tau(E_L^0 - E_L^n) - \mathrm{i}xP_L^n]. \tag{10}$$

Here E_L^0 is the energy of the ground state, E_L^n, P_L^n are the energy and momentum of the intermediate states and $z = v\tau + \mathrm{i}x$, $\bar z = v\tau - \mathrm{i}x$.

Comparing (9) and (10) at τ, L going to infinity we obtain the relation between the scaling dimensions Δ in (9) and the finite-size asymptotics of the low lying levels E^n. Decomposition (7) corresponds to the approximation where the contributions of just the four lowest energy levels E^1, E^2, E^3, E^4 with form factors $\langle 0|\phi(0,0)|n\rangle \neq 0$ ($n = 0, 1, \ldots, 4$) are considered:

$$E_L^1 - E_L^0 = (2\pi v/L)\theta_0 \qquad P_L^1 = (2\pi/L)s_0 \tag{11}$$

$$E_L^2 - E_L^0 = (2\pi v/L)\theta_0 \qquad P_L^2 = (2\pi/L)\tilde s_0 \tag{12}$$

$$E_L^3 - E_L^0 = (2\pi v/L)\theta_1 \qquad P_L^3 = (2\pi/L)s_1 + 2k_{\mathrm F} \tag{13}$$

$$E_L^4 - E_L^0 = (2\pi v/L)\theta_1 \qquad P_L^4 = (2\pi/L)s_1 - 2k_{\mathrm F}. \tag{14}$$

The coefficients A, $\tilde A$ and B in (9) are expressed in terms of corresponding form factors:

$$A = \lim_{L\to\infty} \{(\pi/L)^{-2\theta_0}\exp(\mathrm{i}\pi s_0)\,|\langle 0|\phi|1\rangle|^2\}$$

$$\tilde A = \lim_{L\to\infty} \{(\pi/L)^{-2\theta_0}\exp(\mathrm{i}\pi\tilde s_0)|\langle 0|\phi|2\rangle|^2\}$$

$$B = \lim_{L\to\infty} \{2(\pi/L)^{-2\theta_1}\exp(\mathrm{i}\pi s_1)|\langle 0|\phi|3\rangle|^2\}.$$

The conformal dimensions are obtained from θ and s as follows:

$$\begin{aligned} \Delta_0 &= \tfrac12(\theta_0 + s_0) & \bar\Delta_0 &= \tfrac12(\theta_0 - s_0)\\ \tilde\Delta_0 &= \tfrac12(\theta_0 + \tilde s_0) & \bar{\tilde\Delta}_0 &= \tfrac12(\theta_0 - \tilde s_0)\\ \Delta_1 &= \tfrac12(\theta_1 + s_1) & \bar\Delta_1 &= \tfrac12(\theta_1 - s_1).\end{aligned} \tag{15}$$

3. Bethe ansatz solvable models

Let us turn now to the completely integrable models [12]. In this case equations (11)-(14) allow us to calculate the critical exponents exactly. We shall consider the one-dimensional Bose gas with δ-function interaction and XXZ Heisenberg antiferromagnet chain. The corresponding Hamiltonians are

$$H_{\mathrm{BG}}=\int_0^L (\partial_x\psi^+\partial_x\psi+\varkappa\psi^+\psi^+\psi\psi-h\psi^+\psi)\,\mathrm{d}x \qquad \varkappa>0,\quad h>0$$

$$H_{XXZ}=\sum_{\alpha=1}^{L}(\sigma_\alpha^1\sigma_{\alpha+1}^1+\sigma_\alpha^2\sigma_{\alpha+1}^2+\cos 2\eta\,\sigma_\alpha^3\sigma_{\alpha+1}^3+\tfrac{1}{2}h\sigma_\alpha^3)$$

$$0<2\eta<\pi,\quad 0<h<4(1-\cos 2\eta)$$

where h is the chemical potential for the Bose gas and the external magnetic field for the XXZ magnet and $\varkappa$, η are the corresponding coupling constants. It should be mentioned that the finite-size corrections for the XXZ chain in zero magnetic field has been considered in [13, 14].

These models are solved by means of the Bethe ansatz. The N-particle wavefunction is parametrised by N numbers λ which satisfy the equation [15, 16]

$$Lp_0(\lambda_l)=2\pi I_l-\sum_{\substack{m=1\\m\neq l}}^{N}\Phi(\lambda_l-\lambda_m). \tag{16}$$

Here p_0 is a bare momentum and Φ is a bare scattering phase:

$$p_0^{\mathrm{BG}}(\lambda)=\lambda \qquad p_0^{XXZ}(\lambda)=\mathrm{i}\log\left(\frac{\cosh(\lambda-\mathrm{i}\eta)}{\cosh(\lambda+\mathrm{i}\eta)}\right)$$

$$\Phi^{\mathrm{BG}}(\lambda)=-\pi+\mathrm{i}\log\left(\frac{\lambda+\mathrm{i}\varkappa}{\lambda-\mathrm{i}\varkappa}\right)$$

$$\Phi^{XXZ}(\lambda)=-\pi+\mathrm{i}\log\left(\frac{\sinh(\lambda+2\mathrm{i}\eta)}{\sinh(\lambda-2\mathrm{i}\eta)}\right).$$

The numbers I_l are integer if N is odd and half integer if N is even.

The bare energy of each particle is

$$\varepsilon_0^{\mathrm{BG}}=\lambda^2-h \qquad \varepsilon_0^{XXZ}=h-2\sin^2 2\eta/\cosh(\lambda+\mathrm{i}\eta)\cosh(\lambda-\mathrm{i}\eta).$$

The eigenvalue of the Hamiltonian is equal to the sum of the bare energies of the particles:

$$E_L=\sum_{j=1}^{N}\varepsilon_0(\lambda_j). \tag{17}$$

The total momentum is equal to

$$P_L=\sum_{j=1}^{N}p_0(\lambda_j). \tag{18}$$

Taking the sum of all the equations in (16) we find that

$$LP_L=2\pi\sum_{j=1}^{N}I_j \tag{19}$$

(we have used the fact that $\Phi(-\lambda)=-\Phi(\lambda)$).

The ground states of the models are constructed by filling the Fermi sphere with N elementary particles having negative energies. The Fermi momentum $k_F = \pi N/L \equiv \pi\rho$. In the thermodynamic limit (L, $N \to \infty$) the density ρ is finite.

Consider the particles in the centre-of-mass system, i.e. P_L^0. Equation (19) then implies $\Sigma\, I_l = 0$. If we regard ρ as the function of h, then the ground state is defined by the following equations [17]:

$$2\pi\rho(\lambda) = p_0'(\lambda) + \int_{-\Lambda}^{\Lambda} K(\lambda, \mu)\rho(\mu)\,d\mu \tag{20}$$

$$\varepsilon(\lambda) = \varepsilon_0(\lambda) + \frac{1}{2\pi}\int_{-\Lambda}^{\Lambda} K(\lambda, \mu)\varepsilon(\mu)\,d\mu \tag{21}$$

$$K(\lambda) = \partial\Phi(\lambda)/\partial\lambda.$$

The parameter Λ is defined by the requirement that $\varepsilon(\Lambda) = 0$; $\rho(\lambda)$ is the distribution function ($\rho(\lambda_l) = [L(\lambda_{l+1} - \lambda_l)]^{-1}$) of the particles in the Dirac sea with momentum $p(\lambda)$:

$$p(\lambda) = p_0(\lambda) + \int_{-\Lambda}^{\Lambda} \Phi(\lambda, \mu)\rho(\mu)\,d\mu. \tag{22}$$

It is easy to verify that $p'(\lambda) = 2\pi\rho(\lambda)$ and $p(\Lambda) = k_F$.

The density ρ is equal to

$$\rho = \int_{-\Lambda}^{\Lambda} \rho(\lambda)\,d\lambda \tag{23}$$

and the energy of the ground state is

$$E_L^0 = -L\int_{-\Lambda}^{\Lambda} \varepsilon_0(\lambda)\rho(\lambda)\,d\lambda = -\frac{L}{2\pi}\int_{-\Lambda}^{\Lambda} \varepsilon(\lambda)p_0'(\lambda)\,d\lambda. \tag{24}$$

Elementary excitations over the ground state are constructed by the 'particles' with momenta $|p| > k_F(|\lambda| > \Lambda)$ and by the 'hole' $|p| < k_F(|\lambda| < \Lambda)$. The energy of the excited state is

$$e(\lambda) = |\varepsilon(\lambda)|.$$

In the vicinity of Fermi level k_F one has

$$e(\lambda) = v|p(\lambda) - k_F|$$

where v is the Fermi velocity, $v = \varepsilon'(\Lambda)/p'(\Lambda)$.

It follows from the low-temperature expansion of the bulk free energy [14, 15] that $c = 1$ for the models considered.

First we examine the case of 'uncharged' operators: the current operator $\phi(x, \tau) \equiv j(x, \tau) = \psi^+(x, \tau)\psi(x, \tau)$ for the Bose gas and the operator of the third spin component $\phi(\alpha, \tau) = \sigma_\alpha^3(\tau)$ for the XXZ chain. In this case the number of the excited particles and holes is the same. For large L there are four intermediate states with lowest energy levels. Two of them, $|1\rangle$, $|2\rangle$, belong to excitations of the first kind—the particle and the hole on the same Fermi level. The excitations of the second kind, $|3\rangle$, $|4\rangle$, are for particles and holes on opposite Fermi levels.

To construct excitations of the first kind we must put $I_N \to I_N + 1$ or $I_1 \to I_1 - 1$ into (16) while the other I_j are left unchanged. The momentum of the pair particle-hole (see (19)) is

$$P_L^{1,2} = \pm 2\pi/L. \tag{25}$$

It follows from (11) and (12) that $s_0 = 1$, $\tilde{s}_0 = -1$. The energy difference is

$$E_L^{1,2} - E_L^0 = \varepsilon(\lambda_p) - \varepsilon(\lambda_h) = \varepsilon'(\Lambda)(\lambda_p - \lambda_h).$$

Writing

$$P_L^{1,2} = p(\lambda_p) - p(\lambda_h) = p'(\Lambda)(\lambda_p - \lambda_h)$$

and taking into account (25) we thus obtain

$$E_L^{1,2} - E_L^0 = 2\pi v/L \qquad \theta_0 = 1. \tag{26}$$

The excitation of the second kind can be regarded as the transition of the Fermi sphere: $\pm\Lambda \to \pm\Lambda + \delta (\pm\Lambda \to \pm\Lambda - \delta)$. To obtain such excitations we must change I_j in (16) by writing $I'_j = I_j \pm 1$. Subtracting (16) from the Bethe equation with the new I'_j we find with accuracy $1/L^2$ that

$$\delta = \frac{1}{L} \frac{Z(\Lambda)}{\rho(\Lambda)}. \tag{27}$$

Here Z is the 'dressed charge' function [19]:

$$Z(\lambda) = 1 + \frac{1}{2\pi} \int_{-\Lambda}^{\Lambda} K(\lambda, \mu) Z(\mu)\, d\mu. \tag{28}$$

This function has the simple physical sense $Z(\lambda) = \partial\varepsilon(\lambda)/\partial h$. The momenta of these states are

$$P_L^{3,4} = \frac{2\pi}{L} \sum I'_j = \pm 2\pi N/L = \pm 2k_F \tag{29}$$

and the energies are

$$E_L^{3,4} = -\frac{L}{2\pi} \int_{-\Lambda\pm\delta}^{\Lambda\pm\delta} \varepsilon^{3,4}(\lambda) p'_0(\lambda)\, d\lambda. \tag{30}$$

Here $\varepsilon^{3,4}(\lambda)$ are defined (with accuracy $O(\delta^3)$) by the equations

$$\begin{aligned} &\varepsilon^{3,4}(\lambda) = \varepsilon_0(\lambda) \mp \delta p_0(\lambda) + \frac{1}{2\pi} \int_{-\Lambda}^{\Lambda} K(\lambda, \mu) \varepsilon^{3,4}(\mu)\, d\mu \\ &\varepsilon^{3,4}(\Lambda \pm \delta) = 0 \end{aligned} \tag{31}$$

and can be expressed in the following way:

$$\varepsilon^{3,4}(\lambda) = \varepsilon(\lambda) \mp \delta u(\lambda) - \tfrac{1}{2} \delta^2 \varepsilon'(\Lambda) \{F(\lambda|\Lambda) + F(\lambda|-\Lambda)\} \tag{32}$$

where $u(\lambda)$ is the antisymmetric function

$$u(\lambda) = p_0(\lambda) + \frac{1}{2\pi} \int_{-\Lambda}^{\Lambda} K(\lambda, \mu) u(\mu)\, d\mu$$

and the function F is defined by

$$2\pi F(\lambda|\mu) = K(\lambda, \mu) + \int_{-\Lambda}^{\Lambda} K(\lambda, \nu) F(\nu|\mu)\, d\nu.$$

Substituting (32) into (30) and using the relation

$$2\pi\rho(\lambda)=p_0'(\lambda)+\int_{-\Lambda}^{\Lambda} p_0'(\mu)F(\mu|\lambda)\,\mathrm{d}\mu$$

we obtain

$$\frac{2\pi}{L}E_L^{3,4}=-\int_{-\Lambda}^{\Lambda}\varepsilon^{3,4}(\lambda)p_0'(\lambda)\,\mathrm{d}\lambda+\delta^2\varepsilon'(\Lambda)p_0'(\Lambda)$$
$$=(2\pi/L)E_L^0+\delta^2 2\pi\varepsilon'(\Lambda)\rho(\Lambda).$$

Clearly

$$E_L^{3,4}-E_L^0=(2\pi v/L)Z^2(\Lambda). \tag{33}$$

From (11)-(14), (25), (26), (29) and (33) one obtains the following expression for the time-dependent correlation function of currents:

$$\langle\phi(z,\bar z)\phi(0,0)\rangle\simeq\langle\phi(0,0)\rangle^2+\frac{A}{z^2}+\frac{\tilde A}{\bar z^2}+\frac{B\cos(2k_{\mathrm F}x)}{|z|^{2\theta_1}}. \tag{34}$$

The currents are the Hermitian operators and therefore $\tilde A=\bar A$, $B=\bar B$. The critical exponent is equal to

$$2\theta_1=2Z^2(\Lambda). \tag{34$'$}$$

This is just the formula for the critical exponent originally obtained in [20].

Let us now consider the correlation function of the 'charged' fields, i.e. $\phi=\psi^+$ for the Bose gas and σ^+ for the XXZ chain. The vacuum in both models is uncharged. This means that the correlations $\langle\phi\phi\rangle$ and $\langle\phi^*\phi^*\rangle$ are identically zero. Taking charge conjugation of correlators we obtain that conformal spins of the fields ϕ are equal to zero and

$$\langle\phi^*(z,\bar z)\phi(0,0)\rangle\simeq C/|z|^{2\theta_+}. \tag{35}$$

The oscillating term in the correlator of two charged fields is less than the non-oscillating one in any case and so it is not written above. To calculate Δ in the models under consideration we again use the finite-size correction approach.

The lowest energy excited state created by the operator ϕ is the ground state of the Hamiltonian corresponding to $N+1$ particles. This state possesses zero momentum:

$$P_L^\phi=0. \tag{36}$$

One can easily calculate the energy difference in terms of the magnetic susceptibility:

$$E_L^\phi-E_L^0=L[f_0(\rho+1/L)-h/L-f_0(\rho)]=\frac{1}{2L}\frac{\partial h}{\partial\rho} \tag{37}$$

(see also [13] for $h=0$). Here f_0 is the free energy at zero temperature. It follows from (36) and (37) that the value of the critical exponent θ_+ in (35) is

$$\theta_+=\frac{1}{4\pi v}\frac{\partial h}{\partial\rho}. \tag{38}$$

One can show, using integral equations (20) and (21), that the following relation is valid [21]:

$$\frac{\partial h}{\partial\rho}=\frac{\pi v}{Z^2(\Lambda)}. \tag{39}$$

It results in the following simple relation between the critical exponents θ_+ and θ_1:

$$2\theta_+ = 1/2\theta_1. \tag{40}$$

Thus the hypothesis of [20] is proved.

It should be mentioned that the critical exponents in the XXZ model depend essentially on the magnetic field (this dependence was described in detail in [21]). At zero magnetic field the critical exponents can be calculated explicitly:

$$\theta_1 = \pi/4\eta. \tag{41}$$

This value coincides with the results of [10].

4. Conclusions

We have studied the infrared asymptotics of some correlators in the XXZ and non-linear Schrödinger models using the finite-size corrections approach. It is shown that this method can also be applied to obtain oscillating terms in the asymptotics.

We have considered the case of the integrable models. It is, however, evident that equation (38) for θ_+ and the form of the first two terms of the asymptotics (34) do not depend on the integrability of the system. Using the Landau theory of Fermi liquids one can show that expression (34′) for θ_1 is universal and that equation (40) is valid for any $(1+1)$ model with $c=1$ in the corresponding conformal theory.

Acknowledgments

We thank V E Korepin and L D Faddeev for many helpful discussions.

Note added in proof. Arbitrary low-lying excitations are the linear combinations of the states described above (see (34) and (35)). Hence for the spectrum of the conformal dimensions we have

$$\Delta_{n,m,s} = \tfrac{1}{4}(ny - my^{-1})^2 + s \qquad n, m, s \in \mathbb{Z}_1 \qquad y = 2Z^2(\Lambda).$$

It must be mentioned that the following anisotropy parameter is usually used for the XXZ model: $2\eta = \pi - \mu$.

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